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Equivalence principle for spontaneously broken gauge fields with external sources

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Abstract. It is shown how to construct uniquely the gauge transformation that takes a spontaneously broken vector gauge system with external sources to the nearest possible cross section to a pure gauge field in t'Hooft's gauge. In gravity the equivalence principle amounts to the existence of a privileged system of coordinates, whereas in the case of gauge fields the role of the (locally) inertial frames of reference is played by blind (colourless) local cross sections making both the gauge potentials and the symmetric traceless part of their derivatives vanish at a certain point. Once in the blind cross sections, propagation equations naturally lead us to find a quadratic condition in the field strength which measures the tightness of the non-abelian internal coupling. In the absence of Higgs scalars, fields obeying this condition are shown to belong, in general, to case 2 of the Wang–Yang classification.

1. Introduction

The equivalence principle plays a central role in the formulation and understanding of gravity as a physical theory. Roughly stated the principle says that curved space–time can be viewed as approximately flat in a sufficiently small region.

Locally there are frames of reference without gravity. More precisely events close to a certain point can be specified by normal coordinates with an origin in it such that (at least) the affinities vanish at this point.

This microscopic flatness has been recently used by Bunch and Parker (1979) to introduce a local momentum space representation near any given point in general curved space–time which allowed them to study the ultraviolet divergences of, for instance, the $\lambda\phi^4$ theory on a general curved background.

In the case of gauge vector fields, instead of local inertial frames of references we shall here find locally blind sections in the principal bundle based on the Minkowski space M_4 (or in the associated Euclidean section E_4 if one intends to analyse ultraviolet quantum properties).

Let us remind the reader that a principal fibre bundle over M_4 with group G consists, following Kobayashi and Nomizu (1963), of a manifold P and an action of G on P. In short, G acts smoothly to the right on P without fixed points and M_4 is the quotient space of P by the equivalence relation induced by the action of G, $M_4 = P/G$. The canonical projection $\pi: P \to M_4$ is also smooth and P is locally trivial, in the sense that any $p \in M_4$ has a neighbourhood U_p such that $\pi^{-1}(U_p)$ is isomorphic to $U_p \times G$. The gauge

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potentials are Lie-algebra valued functions (A_μ) defined on M_4, and it is always possible to consider $A_\mu(x)$ as the components of a connection form on the trivial principal fibre bundle $P = M_4 \times G$, associated to a certain section σ (Daniel and Viallet 1980). (A section σ is a map from the base space M_4 to the bundle space P such that $\pi \cdot \sigma(p) = p$ for $p \in M_4$.)

In this context, a gauge transformation $R: M_4 \to G$ can be regarded as acting in $P = M_4 \times G$ such that $R: (p, g) \to (p, gR)$. In particular the gauge transformation transforms a section $\sigma = (p, g(p))$ into another cross section $\sigma_R \equiv (p, g(p)R(p))$ (Trautmann 1970). The components of the new connection form induced by the initial Lie-algebra valued functions (A_μ) on σ_R are given in terms of the A_μ associated with σ by

$$A^R_\mu = RA_\mu R^{-1} - R(R^{-1})_{,\mu} \tag{1}$$

where $R \in SU(2)$ and $(R^{-1})_{,\mu} \equiv \partial_\mu(R^{-1})$.

Sections are in gauge fields the substitutes of the more intuitive local frames of reference of gravitation. They are elements of the bundle space and as one of the constituents of the bundle P, the group G, reflects an internal symmetry of the theory the whole P (and therefore any section of P) does not have an intuitive space–time representation.

In the next section we shall determine a cross section σ_R such that A^R_μ will be the closest possible to a pure gauge as it can be, taking into account the existence of isoscalars and external sources. This cross section of the principal bundle is locally blind. Then, in § 3 we apply the blind section to find a condition for nearly decoupled source equations. Thereafter in § 4 it is analysed whether uniform and Wang–Yang types of gauge fields satisfy the decoupling conditions previously found and finally we discuss the results.

2. Determination of the blind gauge

As fermions do not have relevance in the arguments presented here it will be simpler to consider the gauge system defined by the action

$$I = \int (\mathrm{d}^4x)(-2^{-2} f^a_{\mu\nu}(A) f^a_{\mu\nu}(A) + 2^{-1}(D_\mu\phi)_i(D_\mu\phi)_i - V(\phi) + A^a_\mu j^{\mathrm{ext}\,a}_\mu) \tag{2}$$

where the Lie-algebra valued connections $A_\mu \equiv A^a_\mu X_a$ and field strength $f_{\mu\nu}(A) \equiv f^b_{\mu\nu} X_b$ have respective isocomponents A^a_μ and $f^b_{\mu\nu}$ in a base of antisymmetric generators X_a of the simple group $G = SU(2)$

$$[X_a, X_b] = c_{abc} X_c \qquad X_a = -X^{\mathrm{T}}_a. \tag{3}$$

The structure constants c_{abc} of G are fully antisymmetric ($c_{abc}(SU(2)) = -\varepsilon_{abc}$). Real Higgs scalars ϕ_i transform according to a representation of the group G which, in general, may be reducible, defined by real antisymmetric generators Y_a

$$[Y_a, Y_b] = c_{abc} Y_a \qquad Y_a = -Y^{\mathrm{T}}_a. \tag{4}$$

The potential $V(\phi)$ is a quartic gauge-invariant polynomial, as for instance in Hagiwara and Ovrut (1979), the covariant derivative for scalars is given by $(D_\mu\phi)_i \equiv \partial_\mu\phi_i + A^a_\mu Y_{aij}\phi_j$, and the field strength $f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\nu, A_\mu]$. Consistency of the system constrains the external current to be conserved $D_\mu j^{\mathrm{ext}}_\mu \equiv \partial_\mu j_\mu - [j_\mu, A_\mu] = 0$.

Independent variations of (A_μ, ϕ_i) in the action (2) yield the propagation equations

$$D_\nu f_{\mu\nu} = j^{\mathrm{H}}_\mu + j^{\mathrm{ext}}_\mu \equiv j_\mu \qquad D_\mu D_\mu \phi + V'(\phi) = 0 \tag{5}$$

where $(j^{\mathrm{H}}_\mu)^a \equiv (D_\mu\phi)^{\mathrm{T}} Y_a \phi$.

It will be useful to split scalars in terms of some constant field v and the shifted scalar ϕ: $\phi \equiv v + \tilde{\phi}$. (This decomposition will achieve relevance after spontaneously breaking the symmetry, once v is identified with the vacuum.)

In order to determine the blind gauge, let us take a point $p \in \mathrm{M}_4$ and a neighbourhood U_p of this point. Flat coordinates $x(q)$ of any point $q \in U_p$ are determined by $x(q) = x(p) + y, |y| \ll 1$.

The gauge transformation we are looking for can be expressed as a power series:

$$R(x+y) \equiv \exp(-u^a(x+y)X_a) = \sum_{n=0}^{\infty} (n!)^{-1}(-1)^n (u^{a_1}X_{a_1})\dots(u^{a_n}X_{a_n})(x+y). \tag{6}$$

Once $u^a(x+y)$ is fixed, $R(x+y)$ will be determined and the initial set of fields $(A_\mu, \phi = v + \tilde{\phi})$ can be transformed to their new representation $(B_\mu, \psi = w + \tilde{\psi})$ by their respective transformation laws

$$B_\mu(x+y) = RA_\mu R^{-1} - R(R^{-1})_{,\mu} \tag{7a}$$

$$\psi(x+y) = \exp(-u^a(x+y)Y_a)\phi(x+y) \equiv S(R)\phi. \tag{7b}$$

Our procedure consists in determining the set of functions $u^a(x+y)$ by the whole set of its derivatives $u_{\mu_1\dots\mu_m}$ at the starting point p.

We start assigning $u(p) \equiv u^a(p)X_a$ (in the following we shall omit internal indices) by setting

$$u(p) = 0. \tag{8}$$

We then impose $B_\mu(p) = 0$. According to equations (6) and (7a) we must have

$$u_\mu(p) = A_\mu(p). \tag{9}$$

The second derivatives of u appear in the transformation law of $\partial_\mu A_\nu$ which arises from equation (7a)

$$\partial_\mu B_\nu(p) = \partial_\mu A_\nu(p) - u_{\mu\nu}(p) + 2^{-1}[A_\nu, A_\mu](p). \tag{10a}$$

If we assign to $u_{\mu\nu}(p)$ the value

$$u_{\mu\nu}(p) = 2^{-1}(\partial_\mu A_\nu + \partial_\nu A_\mu)(p) - 2^{-2}\zeta(v^{\mathrm{T}}, Y_a\tilde{\phi})(p)\eta_{\mu\nu} \tag{10b}$$

where ζ is an arbitrary real number, the first derivatives of the potentials in the new representation will take the value

$$\partial_\mu B_\nu(p) = 2^{-1} f_{\mu\nu}(p) + 2^{-2}\zeta(v^{\mathrm{T}}, Y_a\tilde{\phi})\eta_{\mu\nu}. \tag{11}$$

Notice that the choice (10b) for the second derivatives of the Lie-valued parameter u gives a value for the new first derivatives $\partial_\mu B_\nu(p)$ such that: (a) $\partial_\mu B_\mu(p) = \zeta(v^{\mathrm{T}} Y_a \tilde{\phi}) = \zeta(w^{\mathrm{T}} Y_a \tilde{\psi})$, the new representation is in t'Hooft's gauge at the point (p) and (b) the object $\partial_\mu B_\nu(p)$ has a vanishing symmetric traceless part. This condition (b) is the most vanishing condition one may impose on $\partial_\mu B_\nu(p)$ consistent with the former condition (a).

The idea is to proceed in this way, at each step obtaining the 'good' value of $u_{\mu_1\ldots\mu_n}(p)$ such that the value of $\partial_{\mu_1\ldots\mu_n}B_\nu(p)$ they consequently define satisfies, at any stage, the corresponding generalisations of the previous conditions (a) and (b).

In the next step currents begin to appear in the results. Taking two derivatives of equation (7a) one obtains the transformation law for the second derivatives of the gauge potentials at the point p.

$$\partial_{\mu_1\mu_2}B_\nu(p)=\partial_{\mu_1\mu_2}A_\nu(p)-u_{\mu_1\mu_2\nu}(p)+2^{-1}[A_{(\nu},u_{\mu_1\mu_2)}](p)+[\partial_{(\mu_1}A_{|\nu|},A_{\mu_2)}](p)+3^{-1}A_{(\mu_1}A_{\mu_2}A_{\nu)}(p)-A_{(\mu_1}A_{|\nu|}A_{\mu_2)}(p) \tag{12a}$$

where $A_{(\nu},u_{\mu_1\mu_2)}\equiv A_\nu u_{\mu_1\mu_2}+A_{\mu_1}u_{\mu_2\nu}+A_{\mu_2}u_{\nu\mu_1}$. In general curved brackets on the indices indicate minimal symmetrisation.

It is convenient to assign to $u_{\mu_1\mu_2\nu}(p)$ the value

$$u_{\mu_1\mu_2\nu}(p)=2^{-1}[A_{(\nu},u_{\mu_1\mu_2)}]+3^{-1}\partial_{(\mu_1\mu_2}A_{\nu)}+3^{-1}[\partial_{(\mu_1}A_{\mu_2},A_{\nu)}]-6^{-1}\eta_{(\mu_1\mu_2}s_{\nu)} \tag{12b}$$

where

$$s_\nu\equiv\zeta D_\nu(v^{\mathrm{T}}Y_a\tilde{\phi})X_a-3^{-1}j_\nu. \tag{12c}$$

Introduction of $u_{\mu_1\mu_2\nu}(p)$ into equation (12a) gives for the second derivatives of the transformed gauge field B_ν

$$\partial_{\mu_1\mu_2}B_\nu(p)=3^{-1}D_{(\mu_1}f_{\mu_2)\nu}-2^{-1}3^{-2}\eta_{(\mu_1\mu_2}j_{\nu)}+2^{-1}3^{-2}\zeta\eta_{(\mu_1\mu_2}D_{\nu)}(vY\tilde{\phi}). \tag{13}$$

Notice that the choice (12b) for the third derivatives of the parameter u implies

$$\partial_{\mu\nu}B_\nu(p)=\zeta D_\mu(vY\tilde{\phi})=\zeta D_\mu^B(wY\tilde{\psi})=\zeta\partial_\mu(wY\tilde{\psi})(p).$$

This is the first derivative at the point p of t'Hooft's gauge condition, which is exactly the statement of condition (a) at the level of second derivatives.

Again, note the symmetric traceless part $\partial^{\mathrm{t}}_{(\mu_1\mu_2}B_{\nu)}(p)$ of the object $\partial_{\mu_1\mu_2}B_\nu(p)$. This is the most vanishing condition one may impose on $\partial_{\mu_1\mu_2}B_\nu(p)$ together with simultaneously requiring the new field to be in t'Hooft's gauge. (Observe that the vacuum w, the transform v as in equation (7b) in general will not be a constant.)

This procedure can be extended to define any finite derivative $u_{\mu_1\ldots\mu_n\nu}(p)$ of the parameter u fixing the gauge transformation $R=\exp(-u)$. For instance the third derivatives of the transformed field will have the value

$$\partial_{\mu_1\mu_2\mu_3}B_\nu(p)=2^{-4}D_{(\mu_1}D_{\mu_2}f_{\mu_3)\nu}+2^{-1}6^{-1}\eta_{(\mu_1\mu_2}(\tfrac{3}{2}s_{\mu_3\nu)}-\tfrac{1}{8}\eta_{\mu_3\nu)}s_{\alpha\alpha}) \tag{14a}$$

where

$$s_{\mu_1\mu_2}(p)\equiv-2^{-2}D_{(\mu_1}j_{\mu_2)}+2^{-1}\zeta\{D_{(\mu_1}D_{\mu_2)}(vY\tilde{\phi})-[u_{\mu_1\mu_2},(v^{\mathrm{T}}Y\tilde{\phi})]\}. \tag{14b}$$

These values come out after having set in the transformation law of the third-order partial derivatives of the gauge potential

$$u_{\mu_1\mu_2\mu_3\nu}(p)=4^{-1}\partial_{(\mu_1\mu_2\mu_3}A_{\nu)}-2^{-1}6^{-1}\eta_{(\mu_1\mu_2}(\tfrac{3}{2}s_{\mu_3\nu)}-\tfrac{1}{8}\eta_{\mu_3\nu)}s_{\alpha\alpha})+2^{-1}[A_{(\nu},u_{\mu_1\mu_2\mu_3)}]+2^{-2}\zeta[(vY\phi),u_{(\mu_2\mu_3}]\eta_{\mu_1\nu)}-6^{-1}[[u_{(\mu_2\mu_3},A_{(\nu}]A_{\mu_1))}]. \tag{14c}$$

It can be checked that $\partial_{\mu_1\mu_2\mu_3}B_\nu(p)$ as given in equation (14a) satisfies requirements (a) and (b): in fact $\partial_{\mu_1\mu_2\nu}B_\nu(p)=2^{-1}\zeta D_{(\mu_1}D_{\mu_2)}(vY\tilde{\phi})=\zeta\partial_{\mu_1\mu_2}(wY\tilde{\psi})$, which are the second derivatives of t'Hooft's gauge at p, and the symmetric traceless part of the third-order derivatives of the new gauge potential $\partial^{\mathrm{t}}_{(\mu_1\mu_2\mu_3}B_{\nu)}(p)$ vanish at p too.

3. Condition for nearly decoupled field equations

In the new gauge defined by R equations (5) are ($j^B_\mu \equiv R j_\mu R^{-1}$)

$$-\Box B_\mu + \partial_{\mu\nu} B_\nu + [B_\nu, \partial_\nu B_\mu + f_{\nu\mu}(B)] - [B_\mu, \partial_\nu B_\nu] = j^B_\mu. \tag{15}$$

These equations hold in a generic point $q \in U_p$ whose coordinates $x(q)$ are in the vicinity of $x(p)$, $x(q) = x(p) + y$, $|y| \ll 1$.

Therefore $\Box B_\mu(q)$, $\partial_\nu B_\mu(q)$, $f_{\nu\mu}(B)(q)$, $B_\mu(q)$ can be infinitesimally expanded to any order in y in the neighbourhood U_p, in terms of the whole set of derivatives $\{\partial_{\mu_1 \ldots \mu_n} B_\nu(p)\}$ calculated at p.

Expanding up to the second order in y equation (15) becomes

$$\begin{aligned} -\Box B_\mu(q) &+ 3 \times 2^{-2} y^\nu \{[f_{\mu\lambda}(B), f_{\nu\lambda}(B)] + [f_{\mu\nu}(B), \Psi]\}(p) \\ &= j^B_\mu(q) - \zeta D_\mu \Psi(p) - 2^{-1} \zeta y^\nu (D_{(\mu} D_{\nu)} \Psi)(p) + \mathrm{O}(y^2) \end{aligned} \tag{16}$$

where $\Psi \equiv (v^{\mathrm{T}} Y^a \tilde{\phi}) X_a$.

One observes two things. The first is the appearance of an effective longitudinal current of the order of $D_\mu \Psi$ due to the gauge condition we have set $\partial_\mu B_\mu(q) = \zeta(w^{\mathrm{T}} Y \tilde{\psi}) = \zeta \Psi^B(q)$ in the whole neighbourhood U_p.

The second consists in the presence of first-order terms, of a clear non-abelian origin. They are proportional to the ζ-dependent, gauge-invariant, antisymmetric Lie-valued element:

$$\Psi_{\mu\nu} \equiv \zeta [f_{\mu\nu}, \Psi] + [f_{\mu\lambda}, f_{\nu\lambda}]. \tag{17}$$

It is clear from equation (16) that in some way $\Psi_{\mu\nu}$ is a measure of how tight is the non-abelian character of the field. The vanishing of $\Psi_{\mu\nu}$, for instance, gives rise to a propagation equation (16) qualitatively closer to $d \equiv \dim \mathrm{G}$ Maxwell uncoupled fields than any other set of gauge fields. It is then worth investigating the implications of vanishing 'gyroscopicity' $\Psi_{\mu\nu}$ (at least in the simpler case where Higgs scalars do not appear).

In the absence of scalars the vanishing of $\Psi_{\mu\nu}$ reduces to

$$\Psi_{\mu\nu}(\phi = 0) \equiv [f_{\mu\lambda}, f_{\nu\lambda}] = 0. \tag{18}$$

In the Lorentz section ($\eta_{\mu\nu} \equiv (-+++)$) of the physical space, introducing the electric and magnetic components of the field strength $e_i \equiv f_{0i}$ and $b_j \equiv 2^{-1} \varepsilon_{jlm} f_{lm}$ these equations can be written

$$[e_i, e_j] - [b_i, b_j] = 0 = [e_i, b_j] - [e_j, b_i]. \tag{19}$$

In general, self-dual or anti-self-dual fields ($e_j = \pm \mathrm{i} b_j$) will not satisfy these six relations which in such cases reduce to the three equations $[e_i, e_j] = 0$.

It is straightforward to see that the less degenerate solutions of equations (19) have the isospace structure (for $\mathrm{G} = \mathrm{SU}(2)$, $e_i \equiv e^a_i X_a$, $b_j \equiv b^a_j X_a$)

$$\begin{aligned} e^a_1 &= (a, 0, 0) & b^a_1 &= (d_1, d_2, 0) \\ e^a_2 &= (b_1, b_2, 0) & b^a_2 &= (e_1, e_2, 0) \\ e^a_3 &= (c_1, c_2, 0) & b^a_3 &= (f_1, f_2, 0). \end{aligned} \tag{20}$$

4. Uniform fields and Wang–Yang types of gauge fields

Gauge fields have been classified by Wang and Yang (1978) using the complex matrix $\Delta^{ab} \equiv K^{ab} + \mathrm{i}J^{ab}$, $K^{ab} \equiv e^a e^b - b^a b^b$, $J^{ab} \equiv e^a b^b + e^b b^a$. In particular gauge fields like those given by equations (20) give rise to a degenerate classifying matrix Δ:

$$\Delta(20) = \begin{pmatrix} \Delta_{11} & \Delta_{12} & 0 \\ \Delta_{21} & \Delta_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{21}$$

Then one strongly suspects that non-gyroscopic gauge fields will belong to case 2 of the Wang–Yang scheme (which corresponds to Δ having rank 2).

It is obvious that this indeed is the case. In fact the set

$$\begin{aligned} e_1^a &= (a, 0, 0) & b_1^a &= (0, \delta a, 0) \\ e_2^a &= (b_1, -\delta b_1, 0) & b_2^a &= (e_1, \delta b_1, 0) \\ e_3^a &= (c_1, -\gamma\delta b_1, 0) & b_3^a &= (\gamma e_1, \delta c_1, 0) \end{aligned} \tag{22}$$

has, in general, a non-vanishing second-rank determinant

$$\begin{aligned} \Delta_2 \equiv \Delta_{11}\Delta_{22} - \Delta_{12}^2 \\ &= (a^2 + b_1^2 + c_1^2)\{(1+\gamma^2)(1-\delta^2)e_1^2 + \mathrm{i}\delta^2(b_1+\gamma c_1)e_1\} \\ &\quad - 2(b_1+\gamma c_1)^2\delta^2 e_1^2 - \mathrm{i}(b_1+\gamma c_1)e_1^3(1+\gamma^2)\delta^2 \neq 0. \end{aligned} \tag{23}$$

The converse is not true. Taking the canonical realisation for case 2 of Wang and Yang (1978) it is easy to check that not necessarily will any case 2 gauge field have vanishing gyroscopicity.

One may also wonder whether the vanishing of $\Psi_{\mu\nu}$ could be related to the fields being uniform, in the sense of Brown and Weissberger (1979).

There is no connection between uniformity and non-gyroscopicity. For instance, for a uniform gauge field A_μ^a such that $Y_{ab} \equiv A_{a\mu}A_{b\mu}$ is of rank 1 with eigenvalues $\lambda_2 = \lambda_3 = 0$, $\lambda_1 > 0$, equations (18) are satisfied while for a uniform gauge field producing a matrix Y_{ab} of rank 3 with $\lambda_1 < 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ one has that $[e_2b_3] - [b_2e_3] \neq 0$. This shows the existence of uniform gauge fields having a gyroscopic character.

To conclude let us also mention that collinear gauge fields, i.e. fields of the form $A_\mu = a_\mu(x)A$, where a_μ is a four-vector and A a matrix, will also satisfy equation (18). Fields of this sort appear when analysing the gauge copies problem (Deser and Wilczek 1976, Bollini *et al* 1979).

5. Conclusions

It has been shown how to construct a gauge transformation in each neighbourhood of the base space M_4 of a principal fibre bundle (with structure group G) such that the associated local cross section is the nearest possible to a pure gauge element. This is in close analogy with the locally inertial frames of gravity. These cross sections can not locally see the gauge potential. They are locally blind in each finite point of M_4.

As one might have expected, scalars and consistent external currents increase the roughness of the blind section which is always consistent with t'Hooft's gauge condition.

In the absence of scalars and external sources the symmetric components of the whole set of partial derivatives $\partial_{(\mu_1 \ldots \mu_n} B_{\nu)}(p)$ vanish while, in their presence, only the traceless part of this object becomes null.

Looking at the propagation equations in the blind section a gauge-invariant condition of quasi-decoupling (non-gyroscopicity) emerged naturally. It is shown that pure gauge fields of this type (no scalars) are a little bit degenerate; they belong to the case 2 of the Wang–Yang classification which is strictly larger than the class of non-gyroscopic fields.

In the Euclidean section instantons will have, in general, a gyroscopic character.

We have also examined uniform gauge fields. It turned out that they can be either truly gyroscopic or have vanishing gyroscopicity as the above explicit examples show.

Regarding possible applications of these results, it is clear that they constitute a possible approximation scheme to short-distance behaviour, and this, in momentum space, is equivalent to high-energy processes in the quanta associated with the gauge potentials (gluons for instance). Let us mention that a further point which will complete the analysis carried out here, interesting in its own right, is that of the long-distance (asymptotic) behaviour of the gauge system in the vicinity of infinity which might contribute to the understanding of the difficult low-energy behaviour of gauge vector fields.

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